IMSc/94/38

Sept 94

Construction of Yangian algebra through a multi-deformation parameter dependent rational R-matrix

B. Basu-Mallick and P. Ramadevi

The Institute of Mathematical Sciences,

C.I.T.Campus, Taramani,

Madras-600 113, INDIA.

e-mail address: biru, rama@imsc.ernet.in

Abstract

Yang-Baxterising a braid group representation associated with multideformed version of $GL_q(N)$ quantum group and taking the corresponding $q \to 1$ limit, we obtain a rational R-matrix which depends on $\left(1+\frac{N(N-1)}{2}\right)$ number of deformation parameters. By using such rational R-matrix subsequently we construct a multiparameter dependent extension of $Y(gl_N)$ Yangian algebra and find that this extended algebra leads to a modification of usual asymptotic condition on monodromy matrix $T(\lambda)$, at $\lambda \to \infty$ limit. Moreover, it turns out that, there exists a nonlinear realisation of this extended algebra through the generators of original $Y(gl_N)$ algebra. Such realisation interestingly provides a novel $\left(1+\frac{N(N-1)}{2}\right)$ number of deformation parameter dependent coproduct for standard $Y(gl_N)$ algebra.

1 Introduction

In recent years Yangian algebras [1] have attracted much attention in the context of some (1+1)-dimensional integrable field models having nonlocal conserved quantities [2,3]. Moreover, it has been observed that a class of one dimensional quantum spin chains with long ranged interactions also possess this novel Yangian symmetry [4-7]. Such a symmetry enables one to derive closed form expressions for many physical quantities like thermodynamic potential and ground state correlation function [7].

These Yangian algebras can be constructed by using the well known quantum Yang-Baxter equation (QYBE) [1,2]

$$R(\lambda-\mu)\, T_1(\lambda)\, T_2(\mu) \ = \ T_2(\mu)\, T_1(\lambda)\, R(\lambda-\mu)\,, \tag{1.1}$$

where $T_1(\lambda) = T(\lambda)\otimes \mathbf{1}$, $T_2(\mu) = \mathbf{1}\otimes T(\mu)$, and $T(\lambda)$ is a $N\times N$ operator valued matrix. The $R(\lambda-\mu)$ appearing in QYBE is taken as a $N^2\times N^2$ c-number matrix, which rationally depends on the spectral parameters and satisfies the Yang-Baxter equation (YBE) :

$$R_{12}(\lambda-\mu)\, R_{13}(\lambda)\, R_{23}(\mu) \ = \ R_{23}(\mu)\, R_{13}(\lambda)\, R_{12}(\lambda-\mu)\,, \tag{1.2}$$

where we have used the standard notations like $R_{12}(\lambda-\mu) = R(\lambda-\mu)\otimes \mathbf{1}$.

However it may be noticed that, in spite of their wide ranged applications, Yangian algebras are usually defined through only one deformation parameter dependent rational solutions of YBE (1.2). For example, the rational R-matrix which leads to $Y(gl_N)$ Yangian algebra is given by [7]

$$R(\lambda) \ = \ \lambda \sum_{i,j=1}^{N} e_{ii}\otimes e_{jj} \ + \ h \sum_{i,j=1}^{N} e_{ij}\otimes e_{ji}\ , \tag{1.3}$$

where $(e_{ij})_{kl} = \delta_{ik}\delta_{jl}$ and h is the single deformation parameter. At $h\rightarrow 0$ limit, the related $Y(gl_N)$ algebra reduces to a subalgebra of infinite dimensional $gl(N)$ Kac-Moody algebra containing its non-negative modes. On the other hand it is worth noting that there exists a large class of multiparameter dependent quantum groups [8-10], which might

be considered as some deformations of finite dimensional Lie groups and can be defined through the spectral parameterless limit of QYBE (1.1). The R-matrix associated with such multiparameter dependent deformation of $GL(N)$ group is given by

$$R \;=\; q \sum_{i=1}^{N} e_{ii} \otimes e_{ii} \;+\; \sum_{i \neq j} e^{i\alpha_{ij}} \; e_{ii} \otimes e_{jj} \;+\; (q - q^{-1}) \sum_{i<j} e_{ij} \otimes e_{ji} \;, \tag{1.4}$$

where one takes $\alpha_{ij} = -\alpha_{ji}$. Apart from the 'quantum' parameter q, the above R-matrix evidently depends on $\frac{N(N-1)}{2}$ number of independent parameters α_{ij}. So it should be interesting to see whether these multiple deformation parameters can also be incorporated in the rational R-matrix structure (1.3) and investigate the corresponding modification of $Y(gl_N)$ Yangian algebra. In sec.2 of this article we shall briefly review the construction of standard $Y(gl_N)$ algebra and subsequently demonstrate that it is indeed possible to deform such algebraic structure through these $\frac{N(N-1)}{2}$ number of extra parameters.

It may be recalled that the multiparameter dependent quantum groups (e.g. $GL_{p,q}(2)$) are in general difficult to realise through generators of corresponding single parameter dependent version (e.g. $GL_q(2)$). However we surprisingly find that, in contrast to the case of usual quantum groups, there exists a nonlinear realisation of present multiparameter dependent extension of $Y(gl_N)$ algebra through the generators of standard $Y(gl_N)$ algebra. By using such realisation, along with the known representations for standard $Y(gl_N)$ [11-13], one may easily build up the representations for multideformed $Y(gl_N)$ algebra. Moreover, that realisation interestingly allows us to construct a new $\left(1 + \frac{N(N-1)}{2}\right)$ number of deformation parameter dependent coproduct for original $Y(gl_N)$ algebra. We discuss these results in sec.3 of this paper and make some concluding remarks in sec.4.

2 Multiparameter dependent extension of $Y(gl_N)$ Yangian algebra

Let us briefly review some properties of standard $Y(gl_N)$ algebra, before exploring its multiparameter deformed version. This $Y(gl_N)$ algebra can be generated by substituting the rational $R(\lambda)$-matrix (1.3) to QYBE (1.1) :

$$(\lambda - \mu) \left[T^{ij}(\lambda) , T^{kl}(\mu) \right] = h \left(T^{kj}(\mu) T^{il}(\lambda) - T^{kj}(\lambda) T^{il}(\mu) \right), \tag{2.1}$$

where $T^{ij}(\lambda)$ are operator valued elements of matrix $T(\lambda)$. If we assume the usual analyticity property of $T(\lambda)$ and asymptotic condition $T(\lambda) \rightarrow 1$ at $\lambda \rightarrow \infty$ [7], then the elements $T^{ij}(\lambda)$ can be expanded in powers of λ as

$$T^{ij}(\lambda) = \delta_{ij} + h \sum_{n=0}^{\infty} \frac{t_n^{ij}}{\lambda^{n+1}} . \tag{2.2}$$

Plugging the above expansion in eqn. (2.1) and comparing from its both sides the coefficients of same powers in spectral parameters, one may easily express the $Y(gl_N)$ algebra through the modes t_n^{ij} :

$$\left[t_0^{ij} , t_n^{kl} \right] = \delta_{il} t_n^{kj} - \delta_{kj} t_n^{il} , \quad \left[t_{n+1}^{ij} , t_m^{kl} \right] - \left[t_n^{ij} , t_{m+1}^{kl} \right] = h \left(t_m^{kj} t_n^{il} - t_n^{kj} t_m^{il} \right) . \tag{2.3a, b}$$

Moreover, with the help of induction procedure, it is not difficult to verify that the $Y(gl_N)$ algebra (2.3a,b) can be equivalently written through a single relation given by

$$\left[t_n^{ij} , t_m^{kl} \right] = \delta_{il} t_{n+m}^{kj} - \delta_{kj} t_{n+m}^{il} + h \sum_{p=0}^{n-1} \left(t_{m+p}^{kj} t_{n-1-p}^{il} - t_{n-1-p}^{kj} t_{m+p}^{il} \right) . \tag{2.3c}$$

Notice that at the limit $h \rightarrow 0$, eqn. (2.3c) reduces to a subalgebra of $gl(N)$ Kac-Moody algebra containing its non-negative modes. Consequently, this Yangian algebra might be considered as some nonlinear deformation of Kac-Moody algebra through the parameter h. The Casimir operators for $Y(gl_N)$ algebra may also be obtained by constructing the corresponding quantum determinant (δ) [1,7]. Moreover, by imposing the restriction $\delta = 1$,

one can generate the nonabelian $Y(sl_N)$ algebra form this $Y(gl_N)$ algebra. It may be observed further that Yangian algebras form a class of Hopf algebra, for which the operations like coproduct (Δ), antipode (ϵ) can be defined [1]. For example, by using the expression $\Delta T(\lambda) = T(\lambda) \dot{\otimes} T(\lambda)$ as well as eqn. (2.2), it is easy to write down the coproducts for all modes of $T(\lambda)$:

$$\Delta t_0^{ij} = \mathbf{1} \otimes t_0^{ij} + t_0^{ij} \otimes \mathbf{1} \, , \quad \Delta t_n^{ij} = \mathbf{1} \otimes t_n^{ij} + t_n^{ij} \otimes \mathbf{1} + h \sum_{p+q=n-1} \sum_{k=1}^{N} t_p^{ik} \otimes t_q^{kj} \, , \tag{2.4}$$

where $n \in [1, \infty]$ and $p, \ q \in [0, \infty]$.

The mode expansion (2.2) can be equivalently expressed through another set of generators J_n^a as

$$T^{ij}(\lambda) = \delta_{ij} + \sum_{n=0}^{\infty} \frac{1}{\lambda^{n+1}} \left\{ J_n^0 \, \delta_{ij} + \sum_{a=1}^{N^2-1} J_n^a \, (t^a)_{ij} \right\} , \tag{2.5}$$

where t^as are traceless Hermitian matrices corresponding to the fundamental representation of $su(N)$ Lie algebra: $[\ t^a, t^b \ ] = f^{abc} t^c$. Again, the $Y(gl_N)$ algebra might be written in terms of new modes J_n^0 and J_n^a, by substituting (2.5) to eqn.(2.1) and comparing the coefficients of spectral parameters. In particular, the modes J_0^a and J_1^b satisfy the commutation relations

$$[\ J_0^a \ , \ J_0^b \ ] = f^{abc} J_0^c \ , \qquad [\ J_0^a \ , \ J_1^b \ ] = f^{abc} \ J_1^c \ . \tag{2.6}$$

Furthermore, it turns out that, all higher level generators of $Y(gl_N)$ or $Y(sl_N)$ algebra can be realised consistently through these 0 and 1-level generators, provided they satisfy a few additional Serre relations [1,2]. The representation theory of these Yangian algebras has also been studied in the literature [11-13] and found to be closely connected with the degenaracies of wave functions corresponding to some quantum integrable spin chains [4,7].

Now for extending the above discussed $Y(gl_N)$ algebra to the multiparameter deformed case, it is necessary to generalise first the form of related rational $R(\lambda)$-matrix (1.3). For this purpose we may notice that the spectral parameterless R-matrix (1.4), associated with multiparameter dependent deformation of $GL(N)$ group, satisfies the Hecke like condition

[14]

$$R \; - \; \tilde{R} \;\; = \;\; (\, q - q^{-1} \,)\, \mathcal{P} \,, \tag{2.7}$$

where $\tilde{R} \; = \; \mathcal{P} R^{-1} \mathcal{P}$ and $\mathcal{P} \; = \; \sum_{i,j=1}^{N} e_{ij} \otimes e_{ji}$ being the permutation operator. So by following Jones' Yang-Baxterisation prescription related to Hecke algebra, one can easily construct a spectral parameter dependent $R(\lambda)$ matrix as [15]

$$R(\lambda) \;\; = \;\; q^{\frac{\lambda}{h}} \; R \; - \; q^{-\frac{\lambda}{h}} \; \tilde{R} \; ,$$

which would be a solution of YBE (1.2). Substituting the explicit form of R-matrix (1.4) to the above expression one subsequently gets

$$\begin{aligned} R(\lambda) \;\; = \;\; & \Big(\, q^{(1+\frac{\lambda}{h})} - q^{-(1+\frac{\lambda}{h})} \, \Big) \sum_{i=1}^{N} e_{ii} \otimes e_{ii} \; + \; (\, q^{\frac{\lambda}{h}} - q^{-\frac{\lambda}{h}} \,) \sum_{i \neq j} e^{i\alpha_{ij}} \; e_{ii} \otimes e_{jj} \\ & + (q - q^{-1}) \left\{ q^{\frac{\lambda}{h}} \sum_{i<j} e_{ij} \otimes e_{ji} \; + \; q^{-\frac{\lambda}{h}} \sum_{i>j} e_{ij} \otimes e_{ji} \right\} \, . \end{aligned} \tag{2.8}$$

It may be noticed that this type of deformation parameter dependent, trigonometric solution of YBE was considered earlier by Perk and Schultz in the context of solvable vertex models [16]. However, the present way of deriving such solution reveals its close connection to multideformed quantum groups. If we multiply the $R(\lambda)$ matrix (2.8) by a factor $h/(q-q^{-1})$ and then take the $q \rightarrow 1$ limit, that would finally yield a rational solution of YBE given by

$$R(\lambda) \;\; = \;\; \lambda \left(\sum_{i=1}^{N} \; e_{ii} \otimes e_{ii} \;\; + \; \sum_{i \neq j} \; e^{i\alpha_{ij}} \; e_{ii} \otimes e_{jj} \right) \; + \; h \, \mathcal{P} \; . \tag{2.9}$$

Apart from parameter h, the above $R(\lambda)$-matrix evidently depends on $\frac{N(N-1)}{2}$ number of antisymmetric deformation parameters α_{ij}. Moreover, at the limit $\alpha_{ij} = 0$ for all i, j, it reduces to the $R(\lambda)$-matrix (1.3) related to standard $Y(gl_N)$ Yangian algebra.

Having found the multiparameter dependent rational $R(\lambda)$-matrix (2.9), we like to explore at present the corresponding modification of $Y(gl_N)$ algebra. So we substitute $R(\lambda)$-matrix (2.9) to QYBE (1.1) and find that the previous eqn.(2.1) gets deformed to

$$(\lambda - \mu) \Big\{ \, \phi_{ik} \, T^{ij}(\lambda) \, T^{kl}(\mu) \; - \; \phi_{jl} \, T^{kl}(\mu) \, T^{ij}(\lambda) \, \Big\} \; = \; h \, \Big(\, T^{kj}(\mu) \, T^{il}(\lambda) \; - \; T^{kj}(\lambda) \, T^{il}(\mu) \, \Big) \, , \tag{2.10}$$

where the notation $\phi_{ij} = e^{i\alpha_{ij}}$ has been used. Next we attempt to express the above multideformed algebra through the modes of $T^{ij}(\lambda)$. However in this context one curiously observes that the usual mode expansion (2.2), related to asymptotic condition $T(\lambda) \rightarrow 1$ at $\lambda \rightarrow \infty$, is no longer consistent with the algebra (2.10). To avoid this problem we propose a modification of mode expansion (2.2) as

$$T^{ij}(\lambda) \;=\; \delta_{ij}\,\tau^{ii} \,+\, \sum_{n=0}^{\infty} \frac{\hat{t}^{ij}_n}{\lambda^{n+1}} \,, \tag{2.11}$$

where τ^{ii}s ($i \in [1,N]$) are some additional nontrivial generators. By substituting this modified expansion to eqn. (2.10) and equating from its both sides the coefficients of same powers in λ, μ, it is not difficult to obtain the following independent relations among the modes τ^{ii}, $\hat{t}^{ij}_n$:

$$\left[\tau^{ii}\,,\,\tau^{jj}\right] \;=\; 0\,, \qquad \tau^{ii}\,\hat{t}^{jk}_n \;=\; \frac{\phi_{ik}}{\phi_{ij}}\,\hat{t}^{jk}_n\,\tau^{ii}\,, \tag{2.12a,b}$$

$$\phi_{ik}\;\hat{t}^{ij}_0\,\hat{t}^{kl}_n \;-\; \phi_{jl}\;\hat{t}^{kl}_n\,\hat{t}^{ij}_0 \;=\; \delta_{il}\;\hat{t}^{kj}_n\,\tau^{ii} \;-\; \delta_{kj}\;\tau^{kk}\,\hat{t}^{il}_n\,, \tag{2.12c}$$

$$\begin{aligned}\Big[\,\phi_{ik}\;\hat{t}^{ij}_{n+1}\,\hat{t}^{kl}_m \;-\; \phi_{jl}\;\hat{t}^{kl}_m\,\hat{t}^{ij}_{n+1}\,\Big] \;-\; \Big[\,\phi_{ik}\;\hat{t}^{ij}_n\,\hat{t}^{kl}_{m+1} \;-\; \phi_{jl}\;\hat{t}^{kl}_{m+1}\hat{t}^{ij}_n\,\Big] \\ =\; h\,\Big(\,\hat{t}^{kj}_m\,\hat{t}^{il}_n \;-\; \hat{t}^{kj}_n\,\hat{t}^{il}_m\,\Big)\,.\end{aligned} \tag{2.12d}$$

With the help of induction procedure, subsequently we find that the equations (2.12c) and (2.12d) can also be expressed through a single relation given by

$$\begin{aligned}\phi_{ik}\;\hat{t}^{ij}_n\,\hat{t}^{kl}_m \;-\; \phi_{jl}\;\hat{t}^{kl}_m\,\hat{t}^{ij}_n \;=\;& \delta_{il}\;\hat{t}^{kj}_{n+m}\,\tau^{ii} \;-\; \delta_{kj}\;\tau^{kk}\,\hat{t}^{il}_{n+m} \\ &+ h \sum_{p=0}^{n-1} \Big(\,\hat{t}^{kj}_{m+p}\,\hat{t}^{il}_{n-1-p} \;-\; \hat{t}^{kj}_{n-1-p}\,\hat{t}^{il}_{m+p}\,\Big)\,.\end{aligned} \tag{2.12e}$$

Comparing with standard $Y(gl_N)$ algebra (2.3), we notice that the relations (2.12a-e) ($Y_m(gl_N)$ algebra) depend on some extra generators τ^{ii} as well as deformation parameters ϕ_{ij} $(= e^{i\alpha_{ij}})$. Moreover from eqn. (2.12b) it is evident that these τ^{ii}s do not commute with all other elements of the deformed algebra. This fact clearly indicates why the previous mode expansion (2.2), recoverable from (2.11) by fixing $\tau^{ii} = 1$, leads to inconsistencies in the present context. However in the particular limit $\phi_{ij} = 1$ for all i, j, one can consistently

put $\tau^{ii} = 1$ in the relations (2.12a-e). Consequently, at this limit our multiparameter dependent $Y_m(gl_N)$ algebra reduces to its single parameter dependent version (2.3).

In analogy with the case of standard Yangian algebra, one can define the coproduct for $Y_m(gl_N)$ algebra (2.12) by using the relation $\Delta T(\lambda) = T(\lambda)\dot{\otimes}T(\lambda)$ and the modified mode expansion (2.11) :

$$
\begin{aligned}
\Delta\tau^{ii} \ &= \ \tau^{ii}\otimes\tau^{ii} \ , \qquad \Delta\hat{t}_0^{ij} \ = \ \tau^{ii}\otimes\hat{t}_0^{ij} \ + \ \hat{t}_0^{ij}\otimes\tau^{jj} \ , \\
\Delta\hat{t}_n^{ij} \ &= \ \tau^{ii}\otimes\hat{t}_n^{ij} \ + \ \hat{t}_n^{ij}\otimes\tau^{jj} \ + \ h \sum_{p+q=n-1}\sum_{k=1}^{N} \hat{t}_p^{ik}\otimes\hat{t}_q^{kj} \ ,
\end{aligned} \tag{2.13}
$$

where $n \in [1,\infty]$ and $p,\ q \in [0,\infty]$. Notice that the above relations do not explicitly depend on extra parameters α_{ij} and reduce to the coproduct of $Y(gl_N)$ (2.4) under the substitution $\tau^{ii} = 1$. Now it is interesting to enquire whether the present $Y_m(gl_N)$ algebra can also be realised through some 0 and 1-level generators satisfying the Serre relations. Moreover, it might be of physical relevence to build up the representations for this multiparameter deformed Yangian algebra. However, rather than directly approaching to these problems, we shall construct in the following a realisation of $Y_m(gl_N)$ algebra through the generators of $Y(gl_N)$ algebra. The existence of such realisation would automatically imply that the $Y_m(gl_N)$ algebra can also be realised through 0 and 1-level generators of original $Y(gl_N)$ algebra. Moreover, by using that realisation and known representations of $Y(gl_N)$ algebra [11-13], one may easily build up the representations for $Y_m(gl_N)$ algebra.

3 Realisation of $Y_m(gl_N)$ algebra

For constructing a realisation of $Y_m(gl_N)$ algebra (2.12) through the generators of $Y(gl_N)$ satisfying (2.3), let us introduce first another associative algebra ($\tilde{Y}(gl_N)$) which contains the modes τ^i and t_n^{ij} :

$$
[\ \tau^i\ ,\ \tau^j\ ] \ = \ 0\ , \quad \tau^i t_n^{jk} \ = \ e^{\frac{i}{2}(\alpha_{ik}-\alpha_{ij})}\ t_n^{jk}\tau^i \ = \ \sqrt{\frac{\phi_{ik}}{\phi_{ij}}}\ t_n^{jk}\tau^i\ , \tag{3.1a,b}
$$

$$\left[\, t^{ij}_n \, , \, t^{kl}_m \, \right] \; = \; \delta_{il} \, t^{kj}_{n+m} - \delta_{kj} \, t^{il}_{n+m} \; + \; h \sum_{p=0}^{n-1} \left(\, t^{kj}_{m+p} \, t^{il}_{n-1-p} \; - \; t^{kj}_{n-1-p} \, t^{il}_{m+p} \, \right) \, . \qquad (3.1c)$$

Notice that the above defined $\tilde{Y}(gl_N)$ is in some sense intermediate between $Y(gl_N)$ and $Y_m(gl_N)$ algebra. On the one hand this $\tilde{Y}(gl_N)$ contains the modes t^{ij}_n, which evidently generate the $Y(gl_N)$ as a subalgebra. While, on the other hand, it has N number of extra generators τ^i which are much similar to the generators τ^{ii} of multideformed $Y_m(gl_N)$ algebra. By taking advantage of this curious link provided by $\tilde{Y}(gl_N)$, we shall construct in the following a realisation of $Y_m(gl_N)$ algebra through the generators of $\tilde{Y}(gl_N)$. Subsequently we shall demonstrate that this intermediate $\tilde{Y}(gl_N)$ algebra, in turn, can be realised through the standard $Y(gl_N)$ generators. Finally, by combining these two realisations, we would be able to express the $Y_m(gl_N)$ generators in terms of $Y(gl_N)$ generators.

To begin with we observe that the $Y_m(gl_N)$ algebra (2.12) allows a simple realisation through the generators of $\tilde{Y}(gl_N)$ (3.1) as

$$\tau^{ii} \; = \; (\tau^i)^2 \, , \qquad \hat{t}^{ij}_n \; = \; \exp\left(-\frac{i\alpha_{ij}}{2} \right) \, \tau^i \tau^j \, t^{ij}_n \, . \qquad (3.2)$$

For checking the validity of this statement, one may consider the case of eqn. (2.12c) as an example. By substituting in it the realisation (3.2) and then using the commutation relations (3.1a,b), it can be brought in the form

$$\mathcal{F} \, [\, t^{ij}_0 \, , \, t^{kl}_n \,] \; = \; \mathcal{F} \, \left(\, \delta_{il} \, t^{kj}_n \; - \; \delta_{kj} \, t^{il}_n \, \right) \, , \qquad (3.3)$$

where $\mathcal{F} \; = \; \exp \, [\, \frac{i}{2} \, (\, \alpha_{ik} + \alpha_{jk} + \alpha_{lk} + \alpha_{jl} + \alpha_{li} + \alpha_{ji} \,) \,] \, \tau^i \tau^j \tau^k \tau^l$. Now with the help of eqn. (3.1c) one can easily verify that the relation (3.3) is indeed satisfied by the generators of $\tilde{Y}(gl_N)$. It is worth noting that the emergence of common factor $\mathcal{F}$ from both sides of eqn. (3.3) plays a crucial role in the above outlined proof. In a similar way we can check the validity of realisation (3.2) for all algebraic relations appearing in (2.12).

Next, we attempt to construct a realisation of $\tilde{Y}(gl_N)$ algebra through the generators t^{ij}_n associated with $Y(gl_N)$ algebra (2.3). Since the $\tilde{Y}(gl_N)$ algebra already contains $Y(gl_N)$ as

a subalgebra, all we need in this case is to express the extra generators τ^i as some functions of modes t_n^{ij} such that the relations (3.1a,b) would be satisfied. So we make an ansatz for these τ^is in the form

$$\tau^i \;=\; \exp\left(i \sum_{l=1}^{N} \beta_{il} \, t_0^{ll} \right) , \tag{3.4}$$

where β_{il} are some yet undetermined constants. Due to relations $[\, t_0^{ii}, \, t_0^{jj} \,] \;=\; 0$, which are some particular cases of eqn. (2.3a), it is evident that the above defined τ^is are commuting among themselves for arbitrary β_{il}. Furthermore, one may observe from eqn. (2.3a) that the modes t_n^{ij} behave like ladder operators with respect to t_0^{ii} :

$$[\, t_0^{ii} \, , \, t_n^{ij} \,] \;=\; -\, t_n^{ij} \, , \quad [\, t_0^{ii} \, , \, t_n^{ji} \,] \;=\; t_n^{ji} \, , \quad [\, t_0^{ii} \, , \, t_n^{jk} \,] \;=\; 0 \, , \tag{3.5}$$

where $i \neq j \neq k$. By using the relation (3.5) it is not difficult to verify that the ansatz (3.4) would satisfy eqn. (3.1b) provided one takes $\beta_{il} \;=\; \frac{\alpha_{il}}{2}$. Consequently, for this particular value of β_{il} the expression (3.4) yields a realisation of $\tilde{Y}(gl_N)$ algebra through the generators of $Y(gl_N)$ algebra. Finally, by combining the relations (3.2) and (3.4), we obtain a realisation of multideformed $Y_m(gl_N)$ algebra (2.12) through the generators of single parameter dependent $Y(gl_N)$ algebra (2.3) :

$$\tau^{ii} \;=\; \exp\left(i \sum_{l=1}^{N} \alpha_{il} \, t_0^{ll} \right) , \qquad \hat{t}_n^{ij} \;=\; \exp\left(-\frac{i\alpha_{ij}}{2} \;+\; \frac{i}{2} \sum_{l=1}^{N} \left(\alpha_{il} + \alpha_{jl} \right) \, t_0^{ll} \right) t_n^{ij} \, . \tag{3.6}$$

With the help of above realisation and known representations [11-13] of $Y(gl_N)$ algebra, one can easily construct the representations for $Y_m(gl_N)$ algebra. Furthermore, by using such realisation, it is also possible to express all higher level generators of $Y_m(gl_N)$ through the 0 and 1-level generators of $Y(sl_N)$ algebra satisfying the usual Serre relations. However, one may consider eqn. (3.6) as a nonlinear transformation which removes the deformation parameters α_{ij} from the $Y_m(gl_N)$ algebra and maps it to single parameter dependent $Y(gl_N)$ algebra. But it is interesting to observe further that, with the help of same transformation (3.6), the coproduct of $Y_m(gl_N)$ (2.13) induces a novel $\left(1 + \frac{N(N-1)}{2} \right)$ number of deformation

parameter dependent coproduct for standard $Y(gl_N)$ algebra :

$$\Delta t_0^{ij} \;=\; K_{ij} \otimes t_0^{ij} \,+\, t_0^{ij} \otimes K_{ji} \;, \quad \Delta t_n^{ij} \;=\; K_{ij} \otimes t_n^{ij} \,+\, t_n^{ij} \otimes K_{ji} \,+\, h \sum_{p+q=n-1} \sum_{k=1}^{N} \Gamma_{ijk} \, t_p^{ik} \otimes t_q^{kj} \;, \tag{3.7}$$

where we have used the notation

$$K_{ij} \;=\; \exp\left(\frac{i}{2} \sum_{l=1}^{N} (\, \alpha_{il} - \alpha_{jl} \,) \; t_0^{ll} \right) \;, \qquad \Gamma_{ijk} \;=\; e^{\frac{i}{2} \left(\alpha_{ij} + \alpha_{jk} + \alpha_{ki} \right)} \; K_{kj} \otimes K_{ki} \;.$$

By applying eqn. (3.5) it is easy to see that the operators K_{ij} satisfy the relations like

$$K_{ik} \, K_{kj} \;=\; K_{ij} \;, \qquad t_0^{kl} \, K_{ij} \;=\; e^{\frac{i}{2} (\, \alpha_{ik} + \alpha_{kj} + \alpha_{jl} + \alpha_{li} \,)} \; K_{ij} \, t_0^{kl} \;. \tag{3.8}$$

With the help of above relations one can also directly verify that the multiparameter dependent coproduct (3.7) is consistent with $Y(gl_N)$ algebra (2.3). Moreover, at the limit $\alpha_{ij} = 0$ for all i, j, eqn. (3.7) reproduces the known coproduct (2.4) of $Y(gl_N)$. Thus it turns out that, though the transformation (3.6) might be used to remove the deformation parameters α_{ij} from $Y_m(gl_N)$ algebra, these deformation parameters interestingly reappear in the coproduct of resulting $Y(gl_N)$ algebra.

4 Conclusion

Here we present a multiparameter dependent extension of $Y(gl_N)$ Yangian algebra, in analogy with the case of multideformed quantum groups. To achieve this, we first Yang-Baxterise a braid group representation associated with multideformed version of $GL_q(N)$ quantum group and take the necessary $q \rightarrow 1$ limit, for obtaining a rational R-matrix which depends on $\left(1 + \frac{N(N-1)}{2}\right)$ number of deformation parameters. Quantum Yang-Baxter equation corresponding to such rational R-matrix interestingly yields a multiparameter dependent extension of standard $Y(gl_N)$ algebra ($Y_m(gl_N)$ algebra).

Next we try to express this $Y_m(gl_N)$ algebra in a spectral parameter independent form and find that the usual asymptotic condition on monodromy matrix $T(\lambda)$, i.e. $T(\lambda) \rightarrow 1$ at

$\lambda \to \infty$, is no longer appropriate for this purpose. To overcome this problem, we modify such asymptotic condition by introducing N number of extra generators τ^{ii}. Consequently, these extra generators also appear in an intriguing fashion in the resulting spectral parameter independent form of our $Y_m(gl_N)$ algebra. Subsequently we find that there exists a nonlinear realisation of $Y_m(gl_N)$ algebra through the generators of standard $Y(gl_N)$. By using such realisation and known representations of $Y(gl_N)$ algebra, one can easily build up the representations of deformed $Y_m(gl_N)$ algebra. Furthermore, this realisation allows us to construct a novel $\left(1+\frac{N(N-1)}{2}\right)$ number of deformation parameter dependent coproduct for standard $Y(gl_N)$ algebra.

As has been remarked earlier, the multiparameter dependent Yangian algebra considered here is actually related to the rational limit of trigonometric R-matrix originally proposed by Perk and Schultz. However, a q-analogue of standard Yangian algebra can also be constructed from quantum Yang-Baxter equation by directly using a single parameter dependent trigonometric R-matrix [17]. So it should be interesting to explore the multiparameter dependent extension of such q-analogue of Yangian algebra. Moreover, as it is well known, there exists a class of quantum integrable models whose Hamiltonians and other conserved quantities can be derived from some realisations of standard Yangian algebra. So it might be of physical interest to search for new integrable models which would be similarly related to the multiparameter dependent extension of Yangain algebra.

References

1. Drinfeld, V.G. , *Sov. Math. Dokl.* **32**, 254 (1985); 'Quantum Groups', *Proc. ICM., Berkeley* , 798 (1987).

2. Bernard, D., *Comm. Math. Phys.* **137**, 191 (1991); *Saclay Priprint,* No. SPhT-92-134 (1992).

3. Babelon, O. and Bernard, D., *Comm. Math. Phys.* **149**, 279 (1992).

4. Bernard, D., Gaudin, M., Haldane, F.D.M. and Pasquier, V., *J. Phys.* **A26**, 5219 (1993).

5. Sutherland, B. and Shastry, B.S., *Phys. Rev. Lett.* **71**, 5 (1993).

6. Hikami, K. and Wadati, M., *J. Phys. Soc. Japan* **62**, 4203 (1993).

7. Haldane, F.D.M., to appear in the *Proceedings of the 16th Taniguchi Symposium, Japan, 1993,* edited by A. Okiji et. al. (Springer, Berlin, 1994) and references therein.

8. Schirrmacher, A., Wess, J. and Zumino, B., *Z. Phys. C- Particles and Fields* **49**, 317 (1991).

9. Sudbey, A., *J. Phys.* **A23**, L697 (1990).

10. Schirrmacher, A., *Z. Phys. C- Particles and Fields* **50**, 321 (1991).

11. Kirillov, A.N. and Reshetikhin, N.Yu., *Lett. Math. Phys.* **12**, 199 (1986).

12. Chari, V. and Pressley, L., *L'Enseignement Math.* **36**, 267 (1990); *J. Reine Angew. Math.* **417**, 87 (1991).

13. Molev, A.I., *Lett. Math. Phys.* **30**, 53 (1994).

14. Wadati, M., Deguchi, T. and Akutsu, Y., *Phys. Rep.* **180**, 247 (1989)

15. Basu-Mallick, B. and Kundu, K., *J. Phys.* **A25**, 4147 (1992).

16. Perk, J.H.H., Schultz, C.L., *Phys. Lett.* **A84**, 407 (1981).

17. Cherednik, I.V., *Duke Math. J.* **54** , 563 (1987).